ACM Turing Award Winners 1986: John Edward Hopcroft and Robert Endre Tarjan

Venkatesh Raman
The Institute of Mathematical Sciences, Chennai

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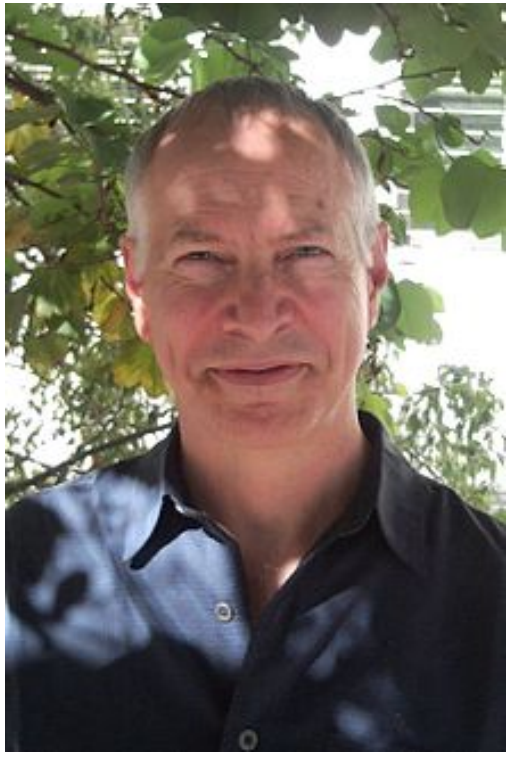

The Turing Award Citation 1986

for fundamental achievements in the design and analysis of algorithms and data structures.

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- Masters from Stanford 1971
- PhD from Stanford 1972 (supervised by Knuth and Floyd, but worked with Hopcroft who was on a sabbatical from Cornell)
- PhD thesis was on a planarity algorithm that won him the Turing Award

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- Graph, algorithms for graph exploration were there (for example, in AI), but no systematic analysis.

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- Efficient Planarity Testing (JACM 1974, IFIP 1971) with John Hopcroft
- Depth First Search and Linear Graph Algorithms (FOCS 1971, SIAM J. Computing 1972)
- Dividing a graph into triconnected components (SIAM J. Computing 1973) with John Hopcroft
- Isomorphism of planar graphs (CCC 1972, JCSS 1973) with John Hopcroft

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- Automata, Complexity theory, Algorithms, Computational Geometry and Robotics, Social Networks, AI, Deep Learning.....

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- ACM Fellow (1994)
- Karl V Karlstrom Outstanding Educator Award (ACM) (2008)
- IEEE John Von Neumann Medal (2010) (with Ullmann)
- *Students* Alfred Aho, Gilles Brassard, Zvi Galil, Cynthia Dwork, 30 of them.

Career (Tarjan)

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- Cornell 1972-73
- UC Berkeley 1973-75
- Stanford 1974-80
- New York University 1981-85
- Princeton since 1985
- Also at Bell Labs (1980-1989), NEC (1989-97), Intertrust technologies (1997-2001, 2014-present), Compaq (2002), HP (2006-2013), Microsoft (2013).

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- Nevanlinna (1983) – the first receipent
- ACM Fellow (1994)
- Paris Kannellakis (1999), ACM – theory and practice (with Sleator for Splay Trees)
- *Students* Daniel Sleator, Samuel Bent, Haim Kaplan, Monika Henzinger, Neil Sarnak, (26 of them)

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- Fast algorithms for network flows (with Goldberg, Ahuja, Orlin, ...)
- Randomized Linear time algorithm for Minimum Spanning Trees (JACM 1995) with Klein and Karger

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- Efficient data structure to support least common ancestor

Book by Tarjan

- Data Structures and Network Algorithms (SIAM publications 1983)

Amortized Complexity, Splay Trees, Dynamic Optimality Conjecture

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- Keeping the elements in a balanced binary search tree makes dictionary queries (insert, search, delete) possible in logarithmic time.
- *All times are worst case*

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- Amortized Cost $\leq$ Worst case cost, but it can be a gross overestimate

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- *Challenge* – Capturing the interdependence of expensive vs cheap operations.

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  Amortized complexity – 2 (one to change a 0 to 1, and one with the new 1).

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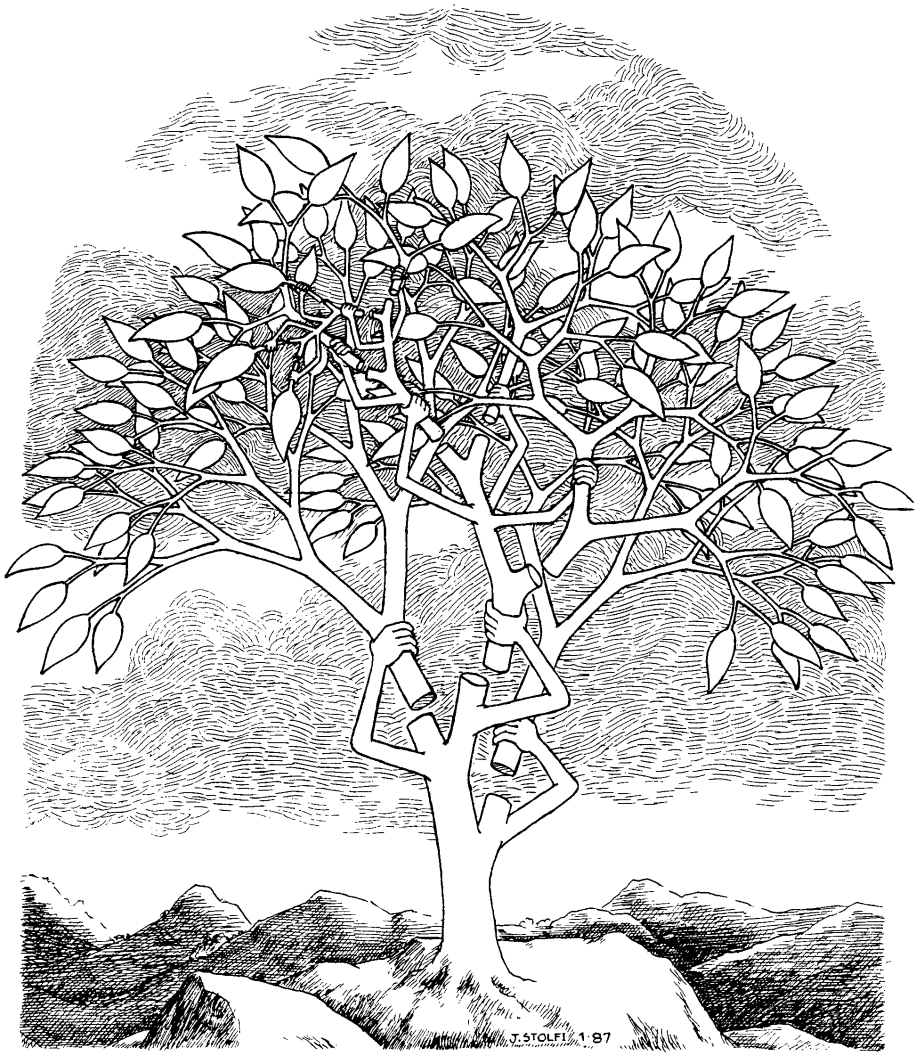
- Dynamic Optimality Conjecture (of Splay Trees)

Self-adjusting trees



A Self-Adjusting Search Tree

Splay Trees – an alternative to balanced binary search trees

Binary Search

7, 13, 18, 90, 201, 207, 400

Search – $O(\log n)$

Insert – $\Theta(n)$

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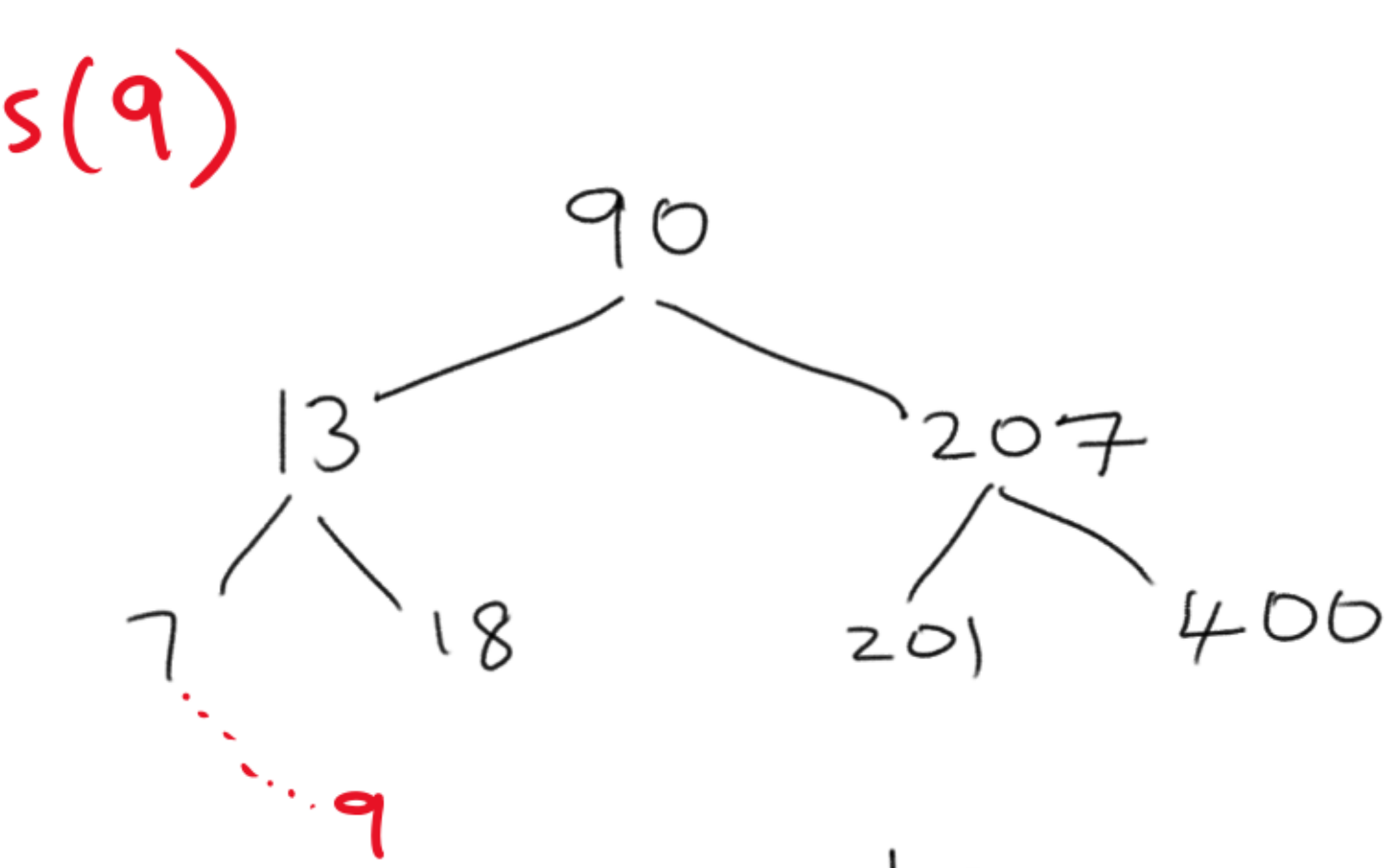

Binary Search tree

Values (left subtree (x)) < Value (x)

< Values (rt subtree of x)

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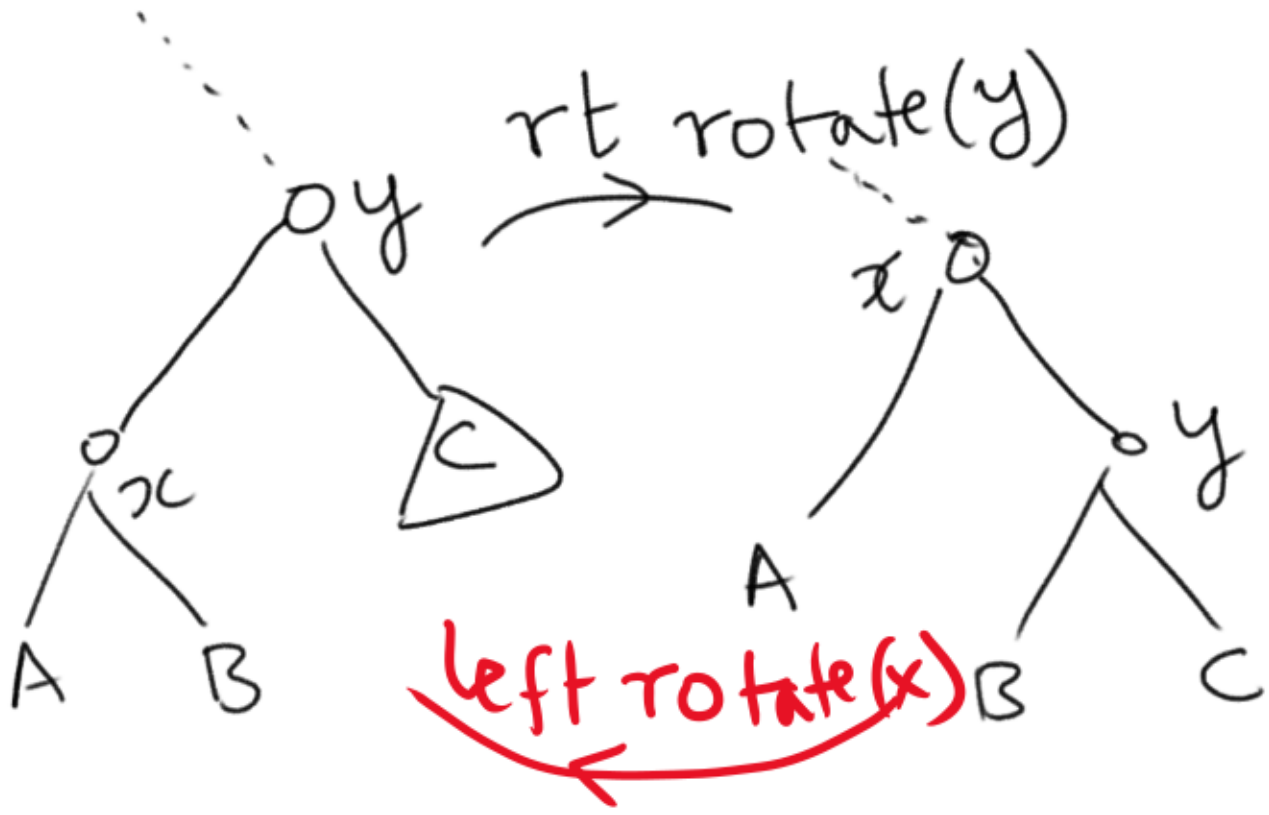
  - Involved case analysis for insertion (and deletion)
  - AVL trees, Red-Black Trees, B-trees, ...

ROTATIONS



- Constant number of pointer changes per rotation
- Binary search tree property maintained

Splay Trees

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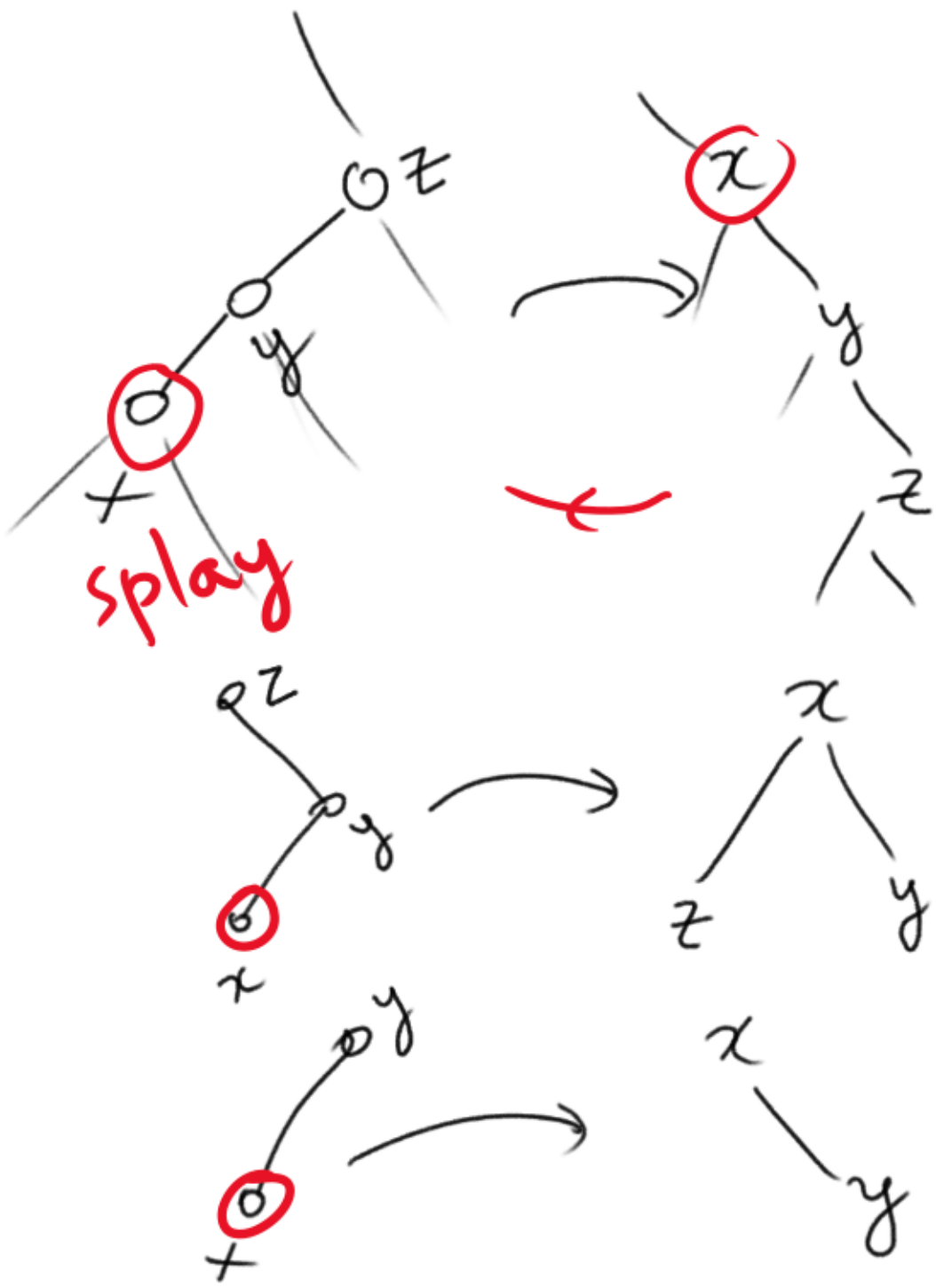
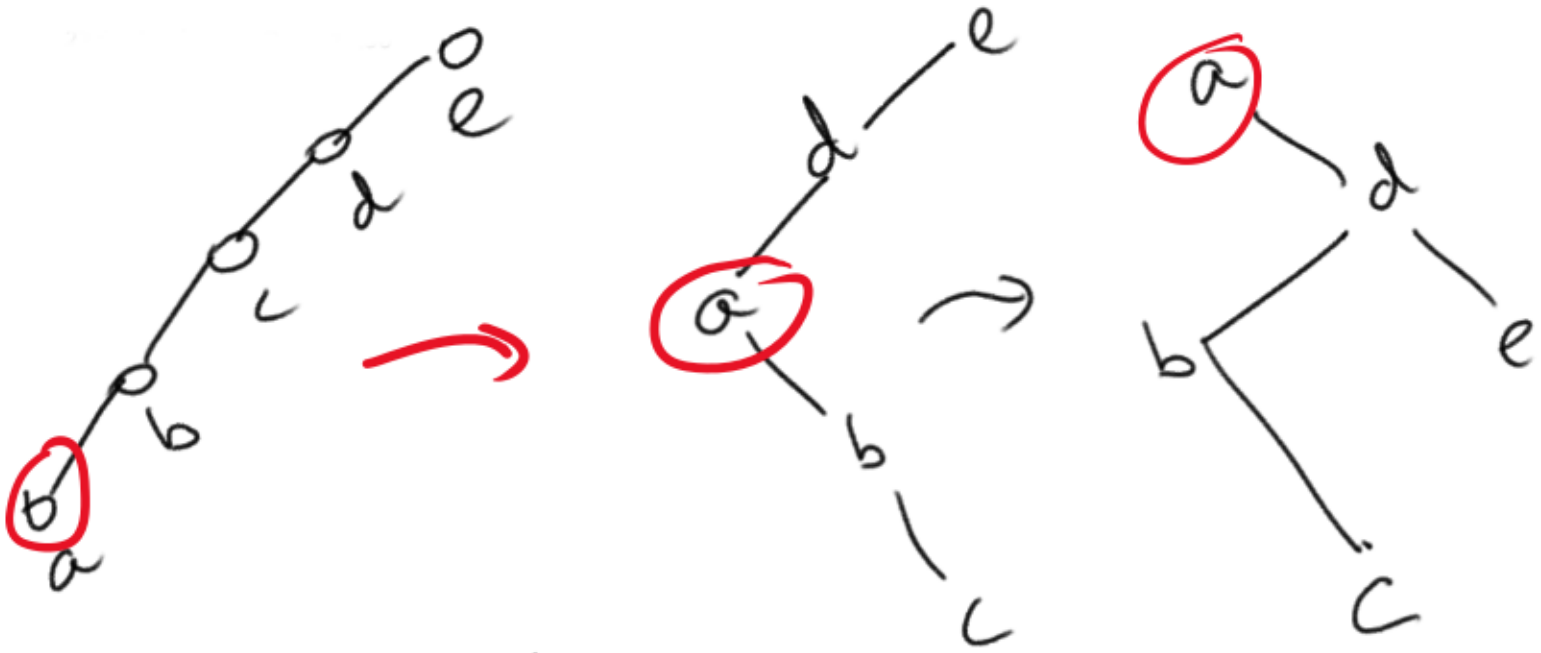
- Independent of the shape of the tree.
- Each splay cuts the depth of each node in the access path by half, and increases the depths of a few others by one or two.
- (Sleator-Tarjan) Amortized Complexity of splaying (hence access/insert/delete) is $O(\log n)$ on any starting tree on n nodes.

SPLAYING





single rotations would have yielded

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  (Static Optimality Theorem of Sleator and Tarjan)
- If the f_is are known, such an optimum binary search tree can be constructed by a dynamic programming based algorithm (Knuth, Cormen et al.)

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- *Dynamic Optimality Conjecture* (Sleator and Tarjan): Total cost of m splay operations on an initial binary search tree on n elements is at most constant times the cost of any other self-adjusting heuristic.

Dynamic Optimality of Move to Front heuristic on lists

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- *Move to Front heuristic:* After accessing an element, move it to the front of the list.
- Theorem (Sleator and Tarjan): For a list, cost of a sequence of accesses using Move to Front heuristic is at most twice the cost of any self-organizing heuristic for any sequence of accesses in an initial list of n elements.

Thank You